Finite Math - Spring 2017

Lecture Notes - 4/14/2017

Homework

- Section 5.1 - 9, 11, 13, 17, 29, 30, 52, 54

Section 5.1 - Linear Inequalities in Two Variables

Graphing Linear Inequalities in Two Variables. There are 4 types of linear inequalities

$$Ax + By \geq C \qquad Ax + By > C$$

$$Ax + By \leq C \qquad Ax + By < C$$

There is a simple procedure to graphing any of these. If equality is not allowed in an inequality, we call it a *strict inequality*, otherwise we simply call it an inequality.

Procedure.

(1) *Graph the line* $Ax + By = C$ *as a dashed line if the inequality is strict. Otherwise, graph it as a solid line.*

(2) *Choose a test point anywhere in the plane, as long as it is not on the line. (The origin,* $(0, 0)$ *is often an easy choice here, but if it is on the line,* $(1, 0)$ *or* $(0, 1)$ *are also easy points to check.)*

(3) *Plug the point from step* (2) *into the inequality. Is the inequality true? Shade in the side of the line with that point. If the inequality is false, shade in the other side.*

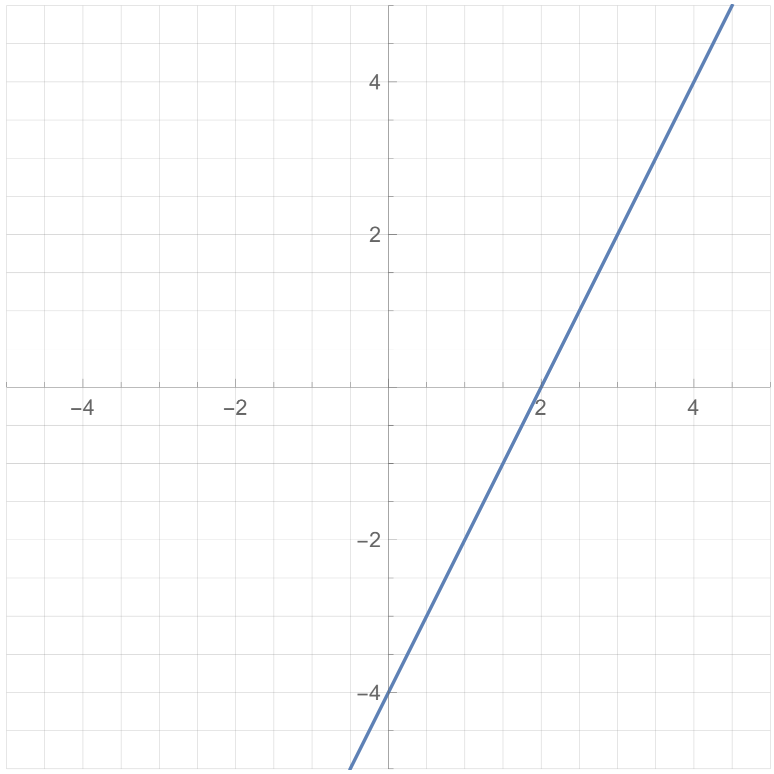
Example 1. *Graph the inequality*

$$6x - 3y \geq 12$$

Solution. *The line we want to graph is*

$$6x - 3y = 12 \quad \text{or} \quad y = 2x - 4.$$

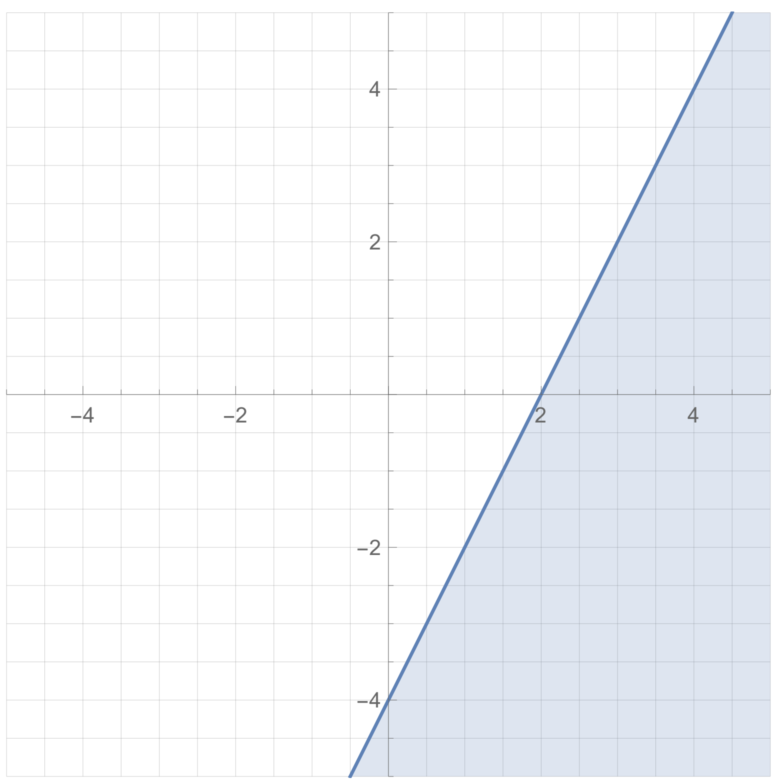
Since the inequality is not strict, we graph it with a solid line.



The point $(0, 0)$ *is not on the line, so we check that point in the inequality*

$$6(0) - 3(0) = 0 \geq 12$$

This is false, so we shade in the side of the line without the origin.



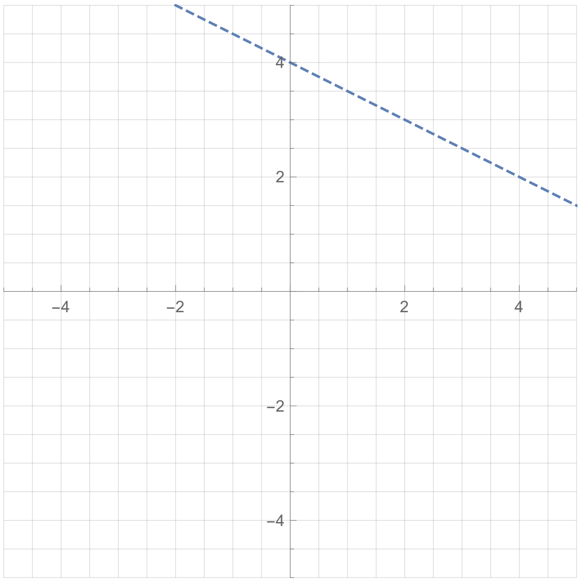
Example 2. *Graph the inequality*

$$4x + 8y < 32$$

Solution. *The line we want to graph is*

$$4x + 8y = 32 \quad \text{or} \quad y = -\frac{1}{2}x + 4.$$

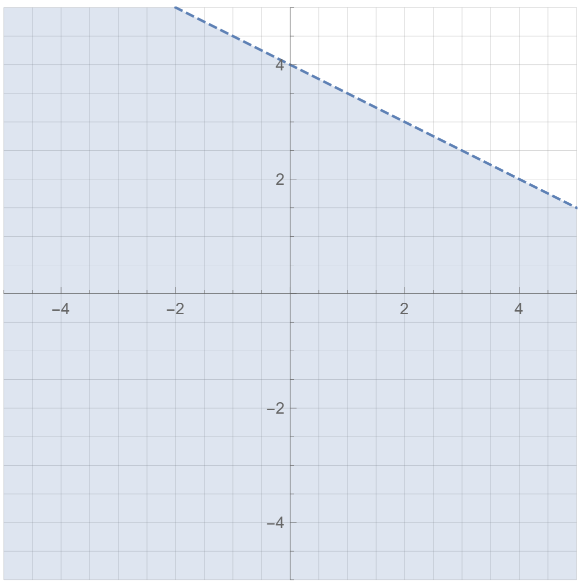
Since the inequality is strict, we graph it with a dashed line.



The point $(0,0)$ is not on the line, so we check that point in the inequality

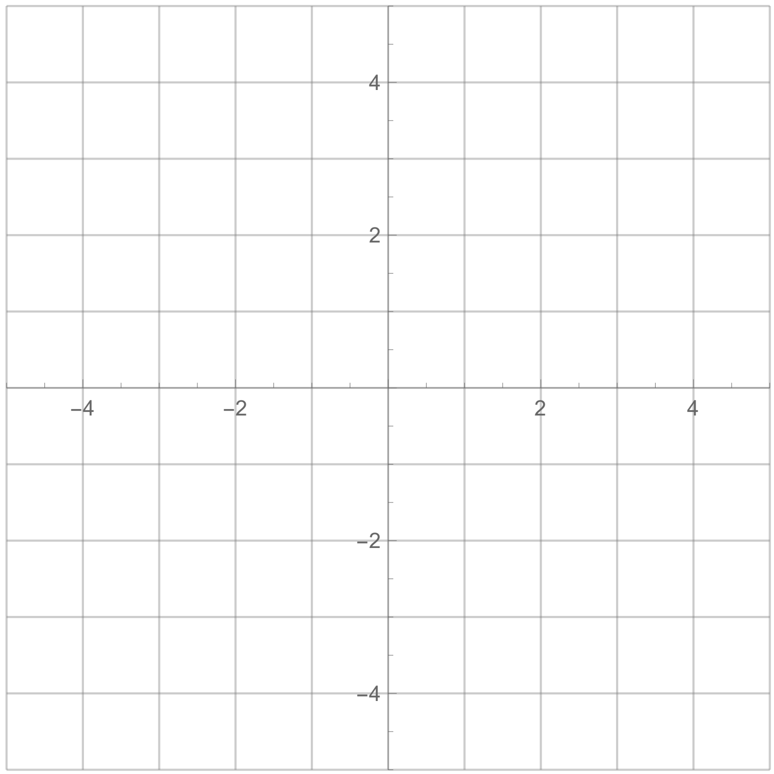
$$4(0) + 8(0) = 0 < 32$$

This is true, so we shade in the side of the line with the origin.



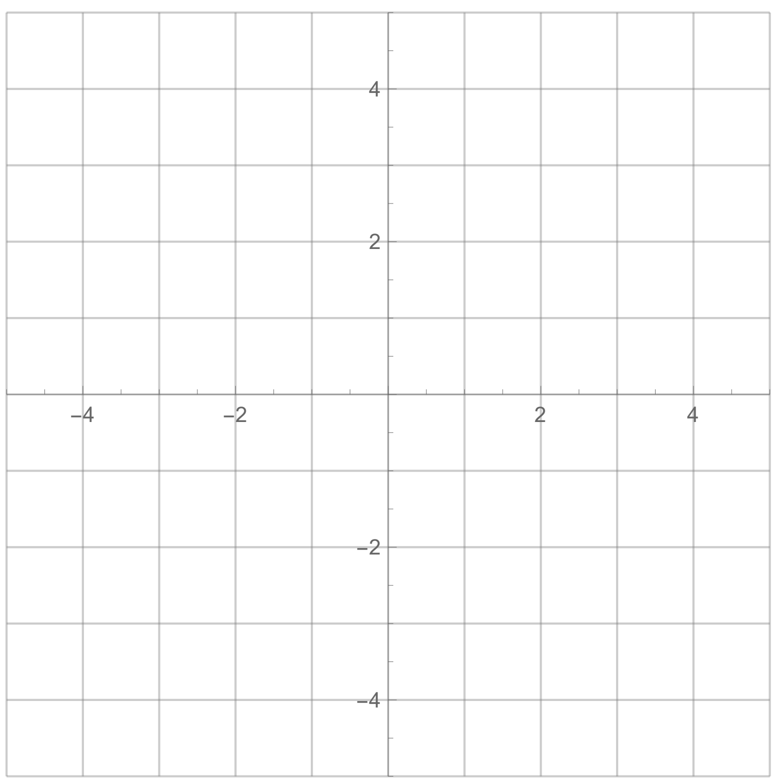
Example 3. *Graph the inequality*

$$2y \leq 10$$

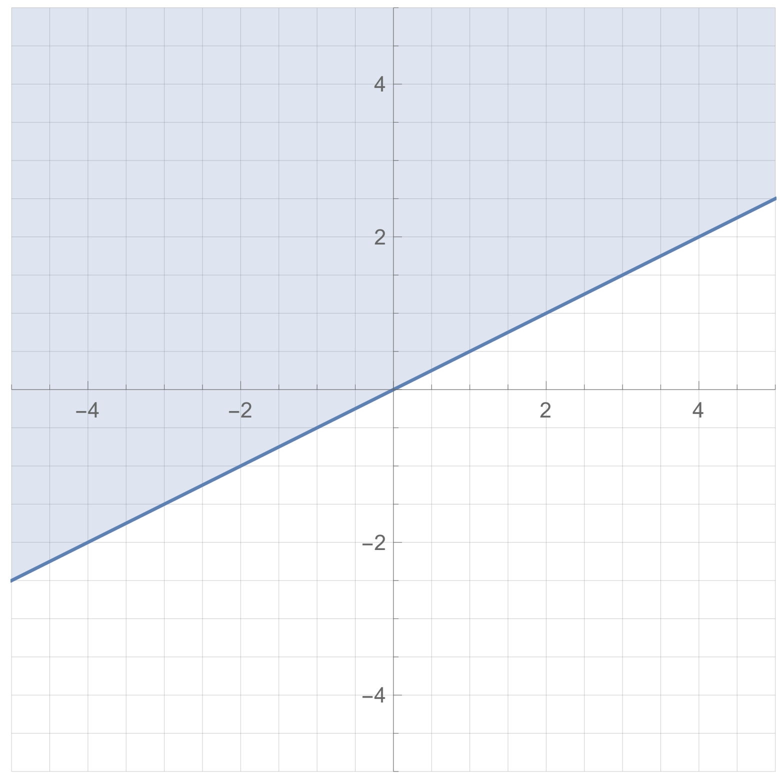


Example 4. *Graph the inequality*

$$2x - 5y > 10$$



Example 5. *Consider the graphed region below.*



(a) *Find an equation for the boundary of the region in the form $Ax + By = C$.*

(b) *Find a linear equality which describes this region.*

Solution.

(a) *Observing the graph, we see that the boundary line passes through $(0, 0)$ and $(2, 1)$. Using the point-slope form, we get*

$$y - 0 = \frac{1 - 0}{2 - 0}(x - 0)$$

which simplifies to

$$y = \frac{1}{2}x$$

Putting this in the required form gives

$$x - 2y = 0.$$

(b) *Because the boundary line is solid, we are going to replace $=$ with either $\geq$ or $\leq$. To figure out which one, we pick a test point which is not on the line and choose the inequality appropriately. If the test point comes from the shaded region, then we pick the inequality which makes the statement true. If the test point comes from outside the shaded region, pick the inequality which makes the statement false.*

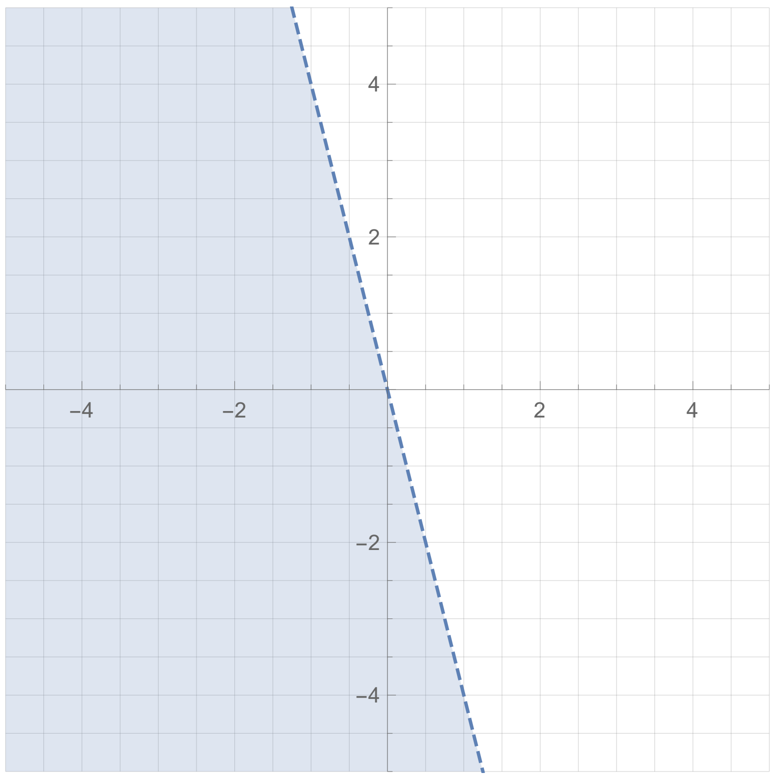
Since $(0,0)$ actually is on this line, we will pick $(1,0)$ as our test point. Notice that $(1,0)$ is outside *the shaded region. Plugging $(1,0)$ into the equation gives*

$$x - 2y = 1 - 2(0) = 1 \boxed{?} 0.$$

Since $(1,0)$ is not in the shaded region, we need to pick the one of $\geq$ and $\leq$ to replace $\boxed{?}$ which makes the statement false. This means we choose $\leq$ giving that the inequality for this picture is

$$x - 2y \leq 0.$$

Example 6. *Consider the graphed region below.*



(a) *Find an equation for the boundary of the region in the form $Ax + By = C$.*

(b) *Find a linear equality which describes this region.*